Preface

Lie theory has its roots in the work of Sophus Lie, who studied certain transformation groups that are now called Lie groups. His work led to the discovery of Lie algebras. By now, both Lie groups and Lie algebras have become essential to many parts of mathematics and theoretical physics. In the meantime, Lie algebras have become a central object of interest in their own right, not least because of their description by the Serre relations, whose generalisations have been very important.

This text aims to give a very basic algebraic introduction to Lie algebras. We begin here by mentioning that "Lie" should be pronounced "lee". The only prerequisite is some linear algebra; we try throughout to be as simple as possible, and make no attempt at full generality. We start with fundamental concepts, including ideals and homomorphisms. A section on Lie algebras of small dimension provides a useful source of examples. We then define solvable and simple Lie algebras and give a rough strategy towards the classification of the finite-dimensional complex Lie algebras. The next chapters discuss Engel's Theorem, Lie's Theorem, and Cartan's Criteria and introduce some representation theory.

We then describe the root space decomposition of a semisimple Lie algebra and introduce Dynkin diagrams to classify the possible root systems. To practice these ideas, we find the root space decompositions of the classical Lie algebras. We then outline the remarkable classification of the finite-dimensional simple Lie algebras over the complex numbers.

The final chapter is a survey on further directions. In the first part, we introduce the universal enveloping algebra of a Lie algebra and look in more detail at representations of Lie algebras. We then look at the Serre relations and their generalisations to Kac–Moody Lie algebras and quantum groups and describe the Lie ring associated to a group. In fact, Dynkin diagrams and the classification of the finite-dimensional complex semisimple Lie algebras have had a far-reaching influence on modern mathematics; we end by giving an illustration of this.

In Appendix A, we give a summary of the basic linear and bilinear algebra we need. Some technical proofs are deferred to Appendices B, C, and D. In Appendix E, we give answers to some selected exercises. We do, however, encourage the reader to make a thorough unaided attempt at these exercises: it is only when treated in this way that they will be of any benefit. Exercises are marked † if an answer may be found in Appendix E and ⋆ if they are either somewhat harder than average or go beyond the usual scope of the text.

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