Vorübungen

Berechnen Sie:

$2^2,\ 2^3,\ 2^4,\ 2^5,\ 2^6,\ 2^{10}$	ergibt:	$4,\ 8,\ 16,\ 32,\ 64,\ 1024$
$3^2,\ 3^4,\ 3^5$		$9,\ 81,\ 243$
$5^2,\ 5^3,\ 5^4$		$25,\ 125,\ 625$
$10^2,\ 10^3,\ 10^4,\ 10^6$		$100,\ 1000,\ 10000,\ 1000000$

$2^3 \cdot 2^{10} = 2^{13}$

(ergibt berechnet:) $= 8 \cdot 1024 = 8192$

$2^6 \cdot 5^6 = 10^6 = 1000000$

also auch

$2^6 \cdot 5^2 \cdot 5^4 = 10^6$

(ergibt berechnet:) $= 64 \cdot 25 \cdot 625 = 1000000$

$(2^5)^2 = 2^{5\cdot 2} = 2^{10}$

also auch $32^2 = 1024$

Positive Exponenten ($\in I\!N$)

Fassen Sie entsprechend der Potenzgesetze zusammen bzw. vereinfachen Sie:

$$x^5y^4z^6x^8y^3z^4 = x^{13}y^7z^{10}$$
$$6x^5y^3z^2x^7y^2z^5 = 6x^{12}y^5z^7$$
$$(x^6)^3 = x^{18}$$
$$(a^2)^3 = a^6$$
$$(-x^3)^4 = x^{12}$$
$$(-x^4)^3 = -x^{12}$$
$$(-x^3)^5 = -x^{15}$$
$$(-2b^5)^4 = 16b^{20}$$
$$(-3c^5)^3 = -27c^{15}$$

Multiplizieren Sie aus und vereinfachen Sie:

$$a^2(a^3b + b^4) = a^5b + a^2b^4$$
$$(a^4 - b)(a^5 + b^5) = a^9 + a^4b^5 - a^5b - b^6$$
$$(u^5 - 3u^2 + 6v^4) \cdot (u^3 - u^3v^4) = u^8 - u^8v^4 - 3u^5 + 3u^5v^4 + 6u^3v^4 - 6u^3v^8$$

$$(a^2-b^2)(a^2+b^2) = a^4-b^4$$
$$(a^3-b^3)(a^3+b^3) = a^6-b^6$$
$$(a^3b^2c-a^5b^6c)a^4b^2 = a^7b^4c-a^9b^8c$$

Klammern Sie so weit wie möglich aus:

$$3x^4y^2z-12x^3y^3z^3+6x^3y^2z^2 = 3x^3y^2z\cdot(x-4yz^2+2z)$$
$$a^6b^5c^2-a^9b^8c = a^6b^5c\cdot(c-a^3b^3)$$

Ganzzahlige Exponenten

Fassen Sie zusammen und kürzen Sie gegebenenfalls:

$$\frac{x^4y^2z^6}{3x^3y^3z^5} = \frac{xz}{3y}$$
$$\frac{u^3v^5-u^6v^4}{u^2v^6-u^5v^5} = \frac{u^3v^4(v-u^3)}{u^2v^5(v-u^3)} = \frac{u}{v}$$
$$4x^3y^8x^{-2}y^{-6} = 4xy^2$$
$$5a^2b^2c^4a^{-2}b^{-2}c^{-3} = 5c$$
$$4u^3v^4u^{-5}v^{-6} = 4u^{-2}v^{-2} = \frac{4}{u^2v^2}$$
$$\left(\frac{-5x^4}{2y^2}\right)^3 = -\frac{125x^{12}}{8y^6} = -\frac{125}{8}x^{12}y^{-6}$$
$$\frac{(u^2v^{-3})^5}{(-2w^2)^4} = \frac{u^{10}v^{-15}}{16w^8} = \frac{u^{10}}{16v^{15}w^8} = \frac{1}{16}u^{10}v^{-15}w^{-8}$$
$$\frac{(a^2-b^2)^2}{(c^{-2})^2} = \frac{a^4-2a^2b^2+b^4}{c^{-4}} = (a^4-2a^2b^2+b^4)c^4 = a^4c^4-2a^2b^2c^4+b^4c^4$$
$$(a^{-5})^3 = a^{-15} = \frac{1}{a^{15}}$$
$$\frac{1}{b^{-c}} = b^c$$
$$(x^{-2})^{-3} = x^6$$
$$(\,(b^{-1})^{-1}\,)^{-1} = b^{-1} = \frac{1}{b}$$
$$\frac{6x^3y^2z^5}{18x^2y^3z^5} = \frac{x}{3y}$$
$$\frac{a^5b^2+a^3b^4}{3a^6b^4+5a^4b^3} = \frac{a^3b^2(a^2+b^2)}{a^4b^3(3a^2b+5)} = \frac{a^2+b^2}{ab(3a^2b+5)}$$

$$\frac{(x^3y - xy^3)^2}{x^8y^4 + x^6y^6} = \frac{(xy)^2(x^2-y^2)^2}{x^6y^4(x^2+y^2)} = \frac{x^4 - 2x^2y^2 + y^4}{x^4y^2(x^2+y^2)} = \frac{x^4 - 2x^2y^2 + y^4}{x^6y^2 + x^4y^4}$$

$$\frac{ac^2 - ad^2}{a^2c^4 - a^2d^4} = \frac{a(c^2-d^2)}{a^2(c^4-d^4)} = \frac{c^2-d^2}{a(c^2-d^2)(c^2+d^2)} = \frac{1}{a(a^2+d^2)}$$

$$\frac{5d^2e^4 - d^2e^2}{15cde^3 - 3cde} = \frac{d^2e^2(5e^2-1)}{3cde(5e^2-1)} = \frac{de}{3c}$$

Multiplizieren Sie aus:

$$(x^2y^{-2} + x^{-2}y^3)x^{-4}y^2 = x^{-2} + x^{-6}y^5 = \frac{1}{x^2} + \frac{y^5}{x^6}$$

$$(x^{n-1}y^{n+1} - xy)x^{1-n}y^{n-1} = y^{2n} - x^{2-n}y^n = y^{2n} - \frac{x^2y^n}{x^n}$$

$$(a^{-2}b^3 - c^4d^{-1})a^2b^{-2}c^{-2}d^2 = bc^{-2}d^2 - a^2b^{-2}c^2d = \frac{bd^2}{c^2} - \frac{ac^2d}{b^2}$$

$$(a^{n-2} - a^{3-2n})a^{n+2} = a^{2n} - a^{5-n} = a^{2n} - \frac{a^5}{a^n}$$

$$(a^{1-n}b^{1+n} + ab^{n-1}) \cdot a^{n+1}b^{1-n} = a^2b^2 + a^{n+2}$$

$$\frac{(-2x^2 \cdot y^{-4})^4}{(-z^3)^5} = \frac{16x^8y^{-16}}{-z^{15}} = -16\frac{x^8}{y^{16}z^{15}}$$

Schreiben Sie ohne negative Exponenten:

$$x^5y^{-4}z^3 = \frac{x^5z^3}{y^4}$$

$$x^{4-n}y^{n-1} = \frac{x^4y^n}{x^ny}$$

$$x^7y^{-8}z^{-2} = \frac{x^7}{y^8z^2}$$

$$\frac{a^7b^{-2}}{c^3d^{-4}} = \frac{a^7d^4}{c^3b^2}$$

$$(a^{-m})^{-n} = a^{m \cdot n}$$

$$\frac{1}{x^{-6}y^2} = \frac{x^6}{y^2}$$

$$x^{n-m}y^{m-n} = \frac{x^ny^m}{x^my^n}$$

$$((u^{-1})^{-1})^{-3} = u^{-3} = \frac{1}{u^3}$$

Kürzen Sie und schreiben Sie ohne negative Exponenten:

$$\frac{x^6 - x^4y^2}{x^3 + x^4z} = \frac{x^4(x^2-y^2)}{x^3(1+xz)} = \frac{x(x^2-y^2)}{1+xz} = \frac{x^3 - xy^2}{1+xz}$$

$$\frac{u^2v^8w^7}{u^3v^6w^2} = \frac{v^2w^5}{u}$$

$$\frac{x^4y^{-6}}{x^{-2}y^8} = \frac{x^6}{y^{14}}$$

$$\frac{4x^8 - 9y^6}{4x^4z + 6y^3z} = \frac{(2x^4-3y^3)(2x^4+3y^3)}{2z(2x^4+3y^3)} = \frac{2x^4-3y^3}{2z}$$

$$\frac{x^5y^2 - 3x^4y^3}{x^6y^3 + x^5y^4} = \frac{x^4y^2(x-3y)}{x^5y^3(x+y)} = \frac{x-3y}{x^2y+xy^2}$$

$$\frac{a^7b^2 + a^9b}{a^{12}b^4 + a^7b} = \frac{a^7b(b+a^2)}{a^7b(a^5b^3+1)} = \frac{b+a^2}{a^5b^3+1}$$

Wurzeln und gebrochene Exponenten

`Beispiele: (alle Basen positiv!)`

$$\sqrt[4]{x^8y^{12}} = \left(x^8y^{12}\right)^{\frac{1}{4}} = x^2y^3$$

$$\sqrt[6]{\frac{a^8b^6}{c^{12}d^{18}}} = \left(\frac{a^8b^6}{c^{12}d^{18}}\right)^{\frac{1}{6}} = \frac{a^{\frac{4}{3}}b}{c^2d^3} = a^{\frac{4}{3}}bc^{-2}d^{-3}$$

$$\sqrt[m]{x\sqrt[n]{y}} = \left(xy^{\frac{1}{n}}\right)^{\frac{1}{m}} = x^{\frac{1}{m}}y^{\frac{1}{n\cdot m}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$$

Beseitigen von Wurzeln im Nenner:

$$\frac{1}{\sqrt{x}} = \frac{1\cdot\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}} = \frac{\sqrt{x}}{x} = \frac{1}{x}\sqrt{x}$$

$$\frac{1}{\sqrt[3]{xy}} = \frac{1\cdot\sqrt[3]{xy}^2}{\sqrt[3]{xy}\cdot\sqrt[3]{xy}^2} = \frac{\sqrt[3]{xy}^2}{xy} = \frac{1}{xy}\sqrt[3]{x^2y^2}$$

$$\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1\cdot\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right)} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2} = \frac{1}{2}\left(\sqrt{5}+\sqrt{3}\right)$$

Schreiben Sie nur mit Exponenten (ohne Wurzeln bzw. Bruchstriche):

$$\sqrt[5]{x^7} = x^{\frac{7}{5}}$$

$$\left(\sqrt[5]{x}\right)^7 = x^{\frac{7}{5}}$$

$$\sqrt[3]{x^6y^9z^2} = \left(x^6y^9z^2\right)^{\frac{1}{3}} = x^2y^3z^{\frac{2}{3}}$$

$$\sqrt[5]{a^2\sqrt[3]{b^4}} = \left(a^2b^{\frac{4}{3}}\right)^{\frac{1}{5}} = a^{\frac{2}{5}}b^{\frac{4}{15}}$$

$$\sqrt{\sqrt{\sqrt{a}}} = \left(\left(a^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = a^{\frac{1}{8}}$$

$$\sqrt{a\sqrt{a\sqrt{a}}} = \left(a\left(a\left(a^{\frac{1}{2}}\right)\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(a\cdot\left(a^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(a\cdot a^{\frac{3}{4}}\right)^{\frac{1}{2}} = \left(a^{\frac{7}{4}}\right)^{\frac{1}{2}} = a^{\frac{7}{8}}$$

$$\frac{1}{\sqrt[3]{ab}} = (ab)^{-\frac{1}{3}} = a^{-\frac{1}{3}}b^{-\frac{1}{3}}$$

$$\sqrt[4]{\frac{x^8y^{12}}{z^{24}}} = \left(\frac{x^8y^{12}}{z^{24}}\right)^{\frac{1}{4}} = \frac{x^2y^3}{z^6} = x^2y^3z^{-6}$$

$$\frac{\sqrt[3]{x^5y^2z^8}}{\sqrt[5]{x^2yz^6}} = \frac{x^{\frac{5}{3}}y^{\frac{2}{3}}z^{\frac{8}{3}}}{x^{\frac{2}{5}}y^{\frac{1}{5}}z^{\frac{6}{5}}} = x^{\frac{5}{3}-\frac{2}{5}}y^{\frac{2}{3}-\frac{1}{5}}z^{\frac{8}{3}-\frac{6}{5}} = x^{\frac{19}{15}}y^{\frac{7}{15}}z^{\frac{22}{15}}$$

$$\frac{\sqrt[n]{x^{n-3}}\left(\sqrt[n]{x}\right)^{2n+1}}{\sqrt[n]{x^{2n-2}}} = \left(\frac{x^{n-3}x^{2n+1}}{x^{2n-2}}\right)^{\frac{1}{n}} = \left(\frac{x^{3n-2}}{x^{2n-2}}\right)^{\frac{1}{n}} = (x^n)^{\frac{1}{n}} = x$$

$$\frac{\sqrt{a^3b^7c^5}}{\sqrt[4]{a^2b^6c^{22}}} = \frac{a^{\frac{3}{2}}b^{\frac{7}{2}}c^{\frac{5}{2}}}{a^{\frac{1}{2}}b^{\frac{3}{2}}c^{\frac{11}{2}}} = a^{\frac{2}{2}}b^{\frac{4}{2}}c^{-\frac{6}{2}} = ab^2c^{-3}$$

$$\frac{\sqrt{a^2-4ab+4b^2}}{a^2-4b^2} = \frac{\left((a-2b)^2\right)^{\frac{1}{2}}}{(a-2b)(a+2b)} = \frac{a-2b}{(a-2b)(a+2b)}$$

$$= \frac{1}{a+2b} = (a+2b)^{-1} \text{ nur, falls } a-2b>0 \text{ gilt!}$$

$$\frac{x}{\sqrt{x}} = x^{1-\frac{1}{2}} = x^{\frac{1}{2}}$$

$$\sqrt[4]{a^6} = a^{\frac{6}{4}} = a^{\frac{3}{2}}$$

$$\left(\sqrt[6]{b^5}\right)^3 = b^{\frac{5\cdot 3}{6}} = b^{\frac{5}{2}}$$

$$\left(\sqrt[4]{a^2b^8c^4}\right)^2 = \left(a^2b^8c^4\right)^{\frac{2}{4}} = \left(a^2b^8c^4\right)^{\frac{1}{2}} = ab^4c^2$$

$$\sqrt[4]{\sqrt[3]{\sqrt{a}}} = \left(\left(a^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{4}} = a^{\frac{1}{2\cdot 3\cdot 4}} = a^{\frac{1}{24}}$$

$$\sqrt[3]{b^2\sqrt[5]{a}} = \left(b^2 a^{\frac{1}{5}}\right)^{\frac{1}{3}} = b^{\frac{2}{3}} a^{\frac{1}{15}}$$

$$\sqrt{a\sqrt{c\sqrt{b}}} = \left(a\left(c\,(b)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = a^{\frac{1}{2}} c^{\frac{1}{4}} b^{\frac{1}{8}}$$

Schreiben Sie als Wurzeln:

$$a^{\frac{2}{7}} = \sqrt[7]{a^2}$$

$$x^{\frac{5}{3}} = \sqrt[3]{x^5}$$

$$b^{0,5} = b^{\frac{1}{2}} = \sqrt{b}$$

$$y^{-\frac{2}{3}} = \frac{1}{y^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{y^2}} = \sqrt[3]{\frac{1}{y^2}}$$

$$c^{\frac{1}{2}} d^{-\frac{3}{2}} = \frac{\sqrt{c}}{\sqrt{d^3}} = \sqrt{\frac{c}{d^3}}$$

$$b^{\frac{9}{8}} = b \cdot b^{\frac{1}{8}} = b \cdot \sqrt[8]{b}$$

$$c^{-1,5} = c^{-\frac{3}{2}} = \frac{1}{c^{\frac{3}{2}}} = \frac{1}{c \cdot \sqrt{c}}$$

$$a^{\frac{3}{4}} b^{\frac{4}{3}} = \sqrt[4]{a^3}\,\sqrt[3]{b^4}$$

$$\left(a^{\frac{2}{3}} x^5\right)^{\frac{1}{3}} = a^{\frac{2}{9}} x^{\frac{5}{3}} = \sqrt[9]{a^2}\,\sqrt[3]{x^5}$$

$$\left(a^{\frac{1}{2}}\right)^{\frac{1}{3}} a^{\frac{1}{3}} = \left(a^{\frac{1}{2}} \cdot a\right)^{\frac{1}{3}} = \left(a^{\frac{3}{2}}\right)^{\frac{1}{3}} = a^{\frac{3\cdot 1}{2\cdot 3}} = a^{\frac{1}{2}} = \sqrt{a}$$

Schreiben Sie unter eine gemeinsame Wurzel:

$$a\sqrt{a} = \sqrt{a^2} \cdot \sqrt{a} = \sqrt{a^3}$$

$$2x\sqrt[3]{y} = \sqrt[3]{(2x)^3} \cdot \sqrt[3]{y} = \sqrt[3]{8x^3 y}$$

$$xy\sqrt[3]{\frac{x}{y}} = \sqrt[3]{x^3y^3}\sqrt[3]{\frac{x}{y}} = \sqrt[3]{x^4y^2}$$

$$\left(\sqrt{5}-\sqrt{4}\right)\sqrt{\sqrt{5}+\sqrt{4}} = \sqrt{\left(\sqrt{5}-\sqrt{4}\right)^2}\sqrt{\sqrt{5}+\sqrt{4}} = \sqrt{\left(\sqrt{5}-\sqrt{4}\right)\left(\sqrt{5}-\sqrt{4}\right)\left(\sqrt{5}+\sqrt{4}\right)}$$

$$= \sqrt{\left(\sqrt{5}-\sqrt{4}\right)\left(\sqrt{5}^2-\sqrt{4}^2\right)} = \sqrt{\left(\sqrt{5}-\sqrt{4}\right)\cdot 1} = \sqrt{\sqrt{5}-2}$$

$$x\sqrt{y} = \sqrt{x^2y}$$

$$\sqrt[3]{b}\cdot\sqrt{d} = b^{\frac{1}{3}}d^{\frac{1}{2}} = b^{\frac{2}{6}}d^{\frac{3}{6}} = \left(b^2d^3\right)^{\frac{1}{6}} = \sqrt[6]{b^2d^3}$$

$$3a\sqrt[5]{a} = \sqrt[5]{3^5a^5a} = \sqrt[5]{243a^6}$$

$$\frac{\sqrt[3]{xy^2}}{x^2yz} = \sqrt[3]{\frac{xy^2}{x^6y^3z^3}} = \sqrt[3]{\frac{1}{x^5yz^3}}$$

$$\sqrt[3]{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[6]{3}$$

$$\sqrt[3]{\sqrt{3}}\cdot\sqrt[5]{\sqrt{3}} = 3^{\frac{1}{6}}\cdot 3^{\frac{1}{10}} = 3^{\frac{1}{6}+\frac{1}{10}} = 3^{\frac{5+3}{30}} = 3^{\frac{8}{30}} = 3^{\frac{4}{15}} = \sqrt[15]{3^4} = \sqrt[15]{81}$$

Vereinfachen Sie:

$$\sqrt[3]{5}\cdot\sqrt[3]{25} = \sqrt[3]{5\cdot 5^2} = \sqrt[3]{5^3} = 5$$

$$\sqrt{\left(\frac{3}{4}\right)^2-\frac{1}{2}} = \sqrt{\frac{9}{16}-\frac{8}{16}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$\sqrt{2}\cdot\sqrt{8} = \sqrt{16} = 4$$

$$\sqrt[3]{\frac{1}{4}+\frac{1}{6}}\cdot\sqrt[3]{\frac{3}{10}} = \sqrt[3]{\frac{(3+2)\cdot 3}{12\cdot 10}} = \sqrt[3]{\frac{1\cdot 1}{4\cdot 2}} = \frac{1}{2}$$

$$\sqrt{12}\cdot\sqrt{\frac{1}{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

$$\sqrt[4]{4}\cdot\sqrt{2} = \left(2^2\right)^{\frac{1}{4}}\cdot 2^{\frac{1}{2}} = 2^{\frac{1}{2}}\cdot 2^{\frac{1}{2}} = 2$$

$$\frac{\sqrt[3]{a^2b}}{\sqrt[4]{a^2b}} = \frac{\left(a^2b\right)^{\frac{1}{3}}}{\left(a^2b\right)^{\frac{1}{4}}} = \left(a^2b\right)^{\frac{1}{3}-\frac{1}{4}} = \left(a^2b\right)^{\frac{1}{12}} = \sqrt[12]{a^2b}$$

$$\frac{\sqrt[5]{x^2y^4z}}{\sqrt[3]{x^4y^2z}} = \frac{x^{\frac{2}{5}}y^{\frac{4}{5}}z^{\frac{1}{5}}}{x^{\frac{4}{3}}y^{\frac{2}{3}}z^{\frac{1}{3}}} = x^{\frac{2}{5}-\frac{4}{3}}y^{\frac{4}{5}-\frac{2}{3}}z^{\frac{1}{5}-\frac{1}{3}}$$

$$= x^{-\frac{14}{15}} y^{\frac{2}{15}} z^{-\frac{2}{15}} = \sqrt[15]{\frac{y^2}{x^{14} z^2}}$$

$$\frac{\sqrt[m]{y^{n-1}}\,\sqrt[m]{y^2}}{\sqrt[2m]{y^2}} = \frac{\left(y^{n-1}y^2\right)^{\frac{1}{m}}}{\left(y^2\right)^{\frac{1}{2m}}} = \frac{\left(y^{n+1}\right)^{\frac{1}{m}}}{y^{\frac{1}{m}}}$$

$$= \left(\frac{y^{n+1}}{y}\right)^{\frac{1}{m}} = \sqrt[m]{y^n} = y^{\frac{n}{m}}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{x^{n+1}}} = \left(\frac{x}{x^{n+1}}\right)^{\frac{1}{n}} = \left(\frac{1}{x^n}\right)^{\frac{1}{n}} = \frac{1}{x}$$

$$\sqrt{2}\cdot\sqrt{32} = \sqrt{64} = 8$$

$$\sqrt[4]{4}\left(\sqrt{\sqrt[3]{2}}\right)^3 = \left(2^2\right)^{\frac{1}{4}} \cdot \left(\left(2^{\frac{1}{3}}\right)^{\frac{1}{2}}\right)^3 = 2^{\frac{1}{2}} \cdot 2^{\frac{1\cdot 1\cdot 3}{3\cdot 2}} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 2$$

$$\sqrt{\left(\frac{5}{2}\right)^2 + 14} = \left(\frac{25}{4} + \frac{14\cdot 4}{4}\right)^{\frac{1}{2}} = \left(\frac{25+56}{4}\right)^{\frac{1}{2}} = \left(\frac{81}{4}\right)^{\frac{1}{2}} = \frac{9}{2}$$

$$\sqrt{\frac{3}{16}}\cdot\sqrt{\frac{1}{3}-\frac{1}{4}} = \frac{1}{4}\left(3\cdot\frac{4-3}{12}\right)^{\frac{1}{2}} = \frac{1}{4}\left(\frac{3}{12}\right)^{\frac{1}{2}} = \frac{1}{4}\cdot\left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$$

$$\frac{\sqrt{4a^2+24ab+36b^2}}{2\sqrt{a^2-9b^2}} = \frac{1}{2}\cdot\left(\frac{(2a+6b)^2}{(a+3b)(a-3b)}\right)^{\frac{1}{2}} = \frac{1}{2}\cdot\left(\frac{2^2(a+3b)^2}{(a+3b)(a-3b)}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\cdot\left(\frac{2^2(a+3b)}{a-3b}\right)^{\frac{1}{2}} = \frac{1}{2}\cdot 2\left(\frac{a+3b}{a-3b}\right)^{\frac{1}{2}} = \sqrt{\frac{a+3b}{a-3b}}$$

Beseitigen Sie die Wurzeln im Nenner:

$$\frac{1}{\sqrt[3]{a}} = \frac{\sqrt[3]{a^2}}{\sqrt[3]{a}\cdot\sqrt[3]{a^2}} = \frac{1}{a}\sqrt[3]{a^2}$$

$$\frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{9}{3}\sqrt{3} = 3\sqrt{3}$$

$$\frac{x}{\sqrt[n]{y^{n-3}}} = \frac{x\cdot\sqrt[n]{y^3}}{\sqrt[3]{y^{n-3}}\cdot\sqrt[3]{y^3}} = \frac{x\cdot\sqrt[n]{y^3}}{\sqrt[n]{y^n}} = \frac{x}{y}\cdot\sqrt[n]{y^3}$$

oder :

$$= \frac{x}{\sqrt[n]{y^n}\cdot\sqrt[n]{y^{-3}}} = \frac{x}{y}\cdot y^{\frac{3}{n}} = \frac{x}{y}\cdot\sqrt[n]{y^3}$$

$$\frac{1}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)} = \frac{1}{x-y}\left(\sqrt{x}+\sqrt{y}\right)$$

$$\frac{1}{\sqrt{a^3}} = \frac{1}{a\sqrt{a}} = \frac{\sqrt{a}}{a\cdot\sqrt{a}\cdot\sqrt{a}} = \frac{1}{a^2}\sqrt{a}$$

$$
\begin{aligned}
\frac{y}{\sqrt[3]{x^2}} &= \frac{y\cdot\sqrt[3]{x}}{\sqrt[3]{x^2}\cdot\sqrt[3]{x}} = \frac{y\sqrt[3]{x}}{\sqrt[3]{x^3}} = \frac{y}{x}\sqrt[3]{x} \\
\frac{x-2}{\sqrt{x}+\sqrt{2}} &= \frac{(x-2)\left(\sqrt{x}-\sqrt{2}\right)}{\left(\sqrt{x}+\sqrt{2}\right)\left(\sqrt{x}-\sqrt{2}\right)} = \frac{(x-2)\left(\sqrt{x}-\sqrt{2}\right)}{\sqrt{x}^2-\sqrt{2}^2} \\
&= \frac{(x-2)\left(\sqrt{x}-\sqrt{2}\right)}{x-2} = \sqrt{x}-\sqrt{2} \\
\frac{\sqrt[n]{y}}{\sqrt[n]{y^{2n-1}}} &= \left(\frac{y}{y^{2n-1}}\right)^{\frac{1}{n}} = \left(\frac{y^2}{y^{2n}}\right)^{\frac{1}{n}} = \frac{1}{y^2}\cdot\sqrt[n]{y^2}
\end{aligned}
$$